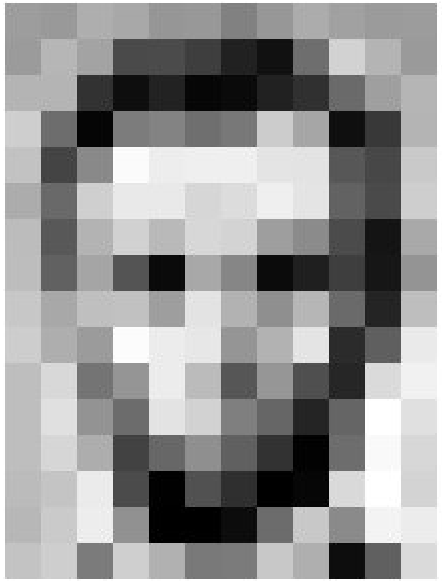
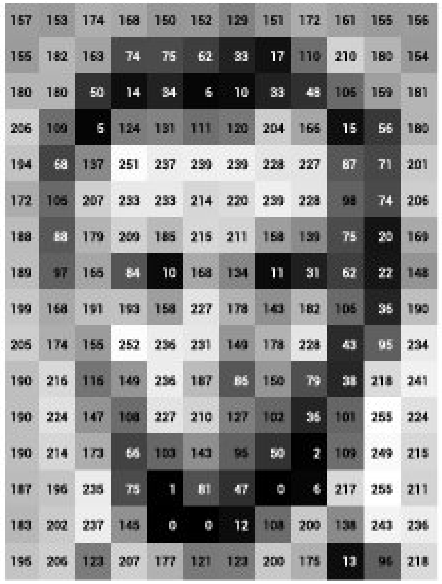
# Linear Image filtering





| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 157 | 153 | 174 | 168 | 150 | 152 | 129 | 151 | 172 | 161 | 155 | 156 |
| 155 | 182 | 163 | 74 | 75 | 62 | 33 | 17 | 110 | 210 | 180 | 154 |
| 180 | 180 | 50 | 14 | 34 | 6 | 10 | 33 | 48 | 106 | 159 | 181 |
| 206 | 109 | 5 | 124 | 131 | 111 | 120 | 204 | 166 | 15 | 56 | 180 |
| 194 | 68 | 137 | 251 | 237 | 239 | 239 | 228 | 227 | 87 | 71 | 201 |
| 172 | 105 | 207 | 233 | 233 | 214 | 220 | 239 | 228 | 98 | 74 | 206 |
| 188 | 88 | 179 | 209 | 185 | 215 | 211 | 158 | 139 | 75 | 20 | 169 |
| 189 | 97 | 165 | 84 | 10 | 168 | 134 | 11 | 31 | 62 | 22 | 148 |
| 199 | 168 | 191 | 193 | 158 | 227 | 178 | 143 | 182 | 106 | 36 | 190 |
| 205 | 174 | 155 | 252 | 236 | 231 | 149 | 178 | 228 | 43 | 95 | 234 |
| 190 | 216 | 116 | 149 | 236 | 187 | 86 | 150 | 79 | 38 | 218 | 241 |
| 190 | 224 | 147 | 108 | 227 | 210 | 127 | 102 | 36 | 101 | 255 | 224 |
| 190 | 214 | 173 | 66 | 103 | 143 | 96 | 50 | 2 | 109 | 249 | 215 |
| 187 | 196 | 235 | 75 | 1 | 81 | 47 | 0 | 6 | 217 | 255 | 211 |
| 183 | 202 | 237 | 145 | 0 | 0 | 12 | 108 | 200 | 138 | 243 | 236 |
| 195 | 206 | 123 | 207 | 177 | 121 | 123 | 200 | 175 | 13 | 96 | 218 |

https://liaoux.github.io/uir-cv-2026/

# Overview of today’s lecture

- What is an image?
- Types of image transformations.
- Point image processing.
- Linear shift-invariant systems.
- Convolution.
- Linear image filters

# What is an image?

# What is an image?

# What is an image?

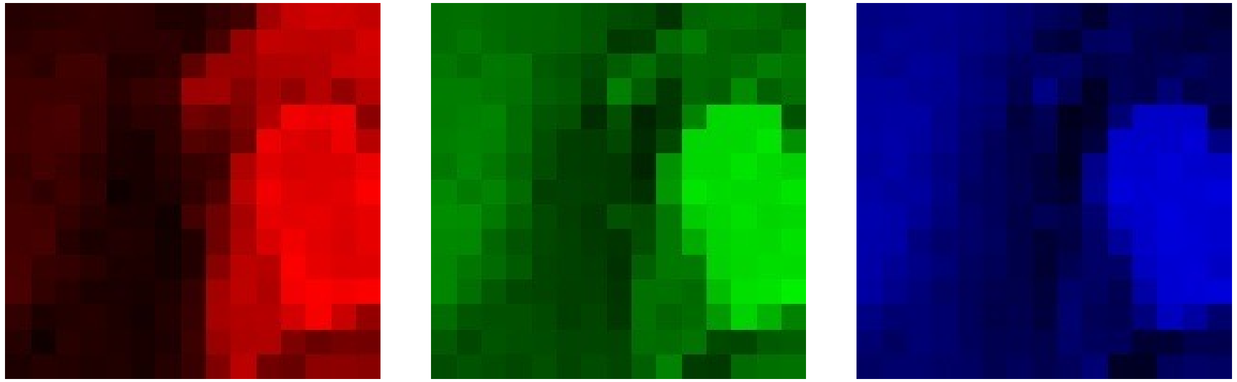
A (color) image is a 3D tensor of numbers.

# Image as a Function

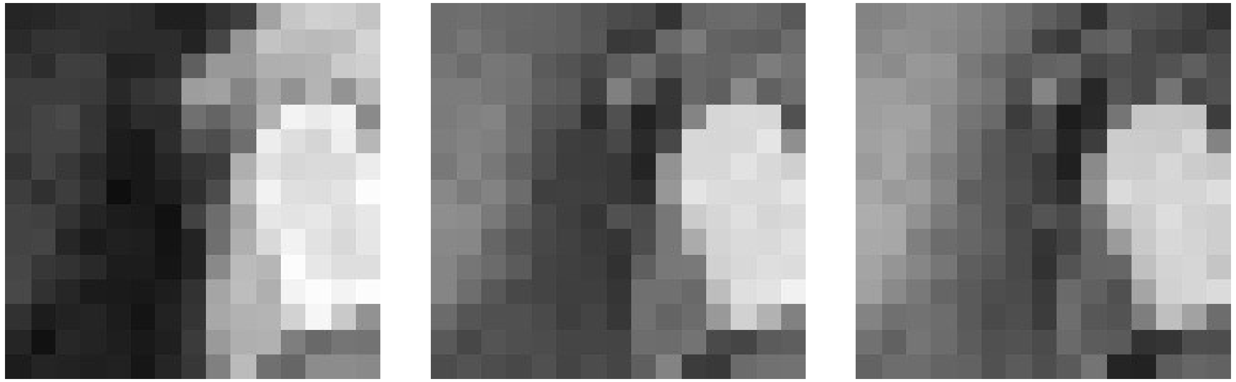

$f(x,y)$ is the image intensity at the spatial coordinates (or position) $(x, y)$

# A color image

red green blue

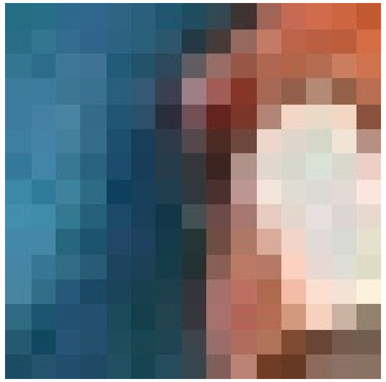


colorized for visualization



actual intensity values per channel

Each channel is a 2D array of numbers.



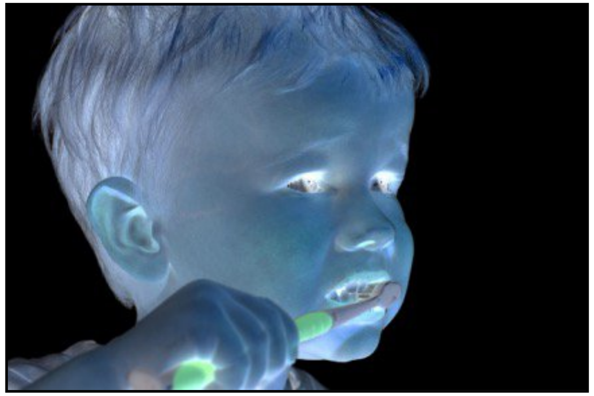
color image patch

How many bits are the intensity values?

# Types of image transformations

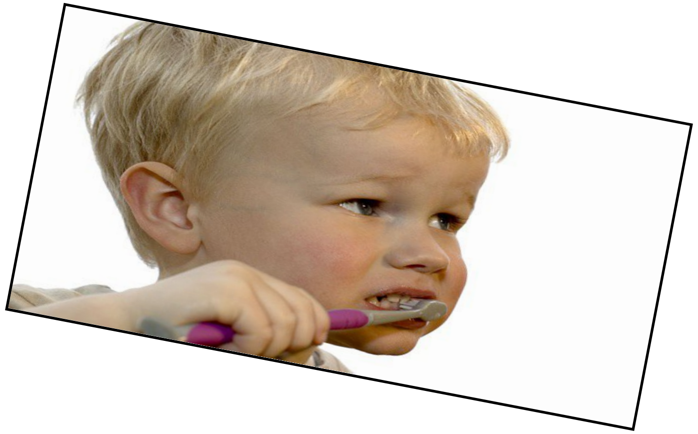
# What types of image transformations can we do?

Filtering



changes pixel values

Warping



changes pixel locations

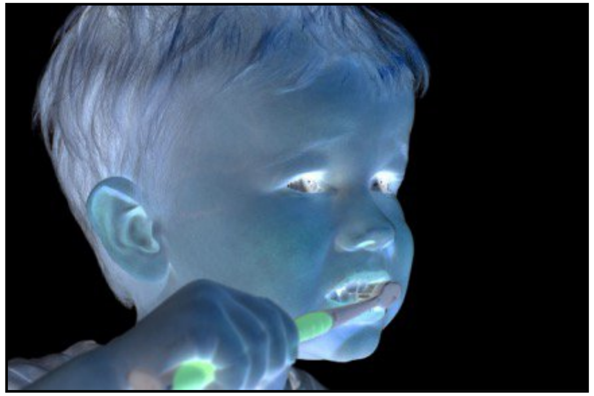
# What types of image transformations can we do?

$F$

Filtering

$$G(\boldsymbol{x}) = h\{F(\boldsymbol{x})\}$$

$G$



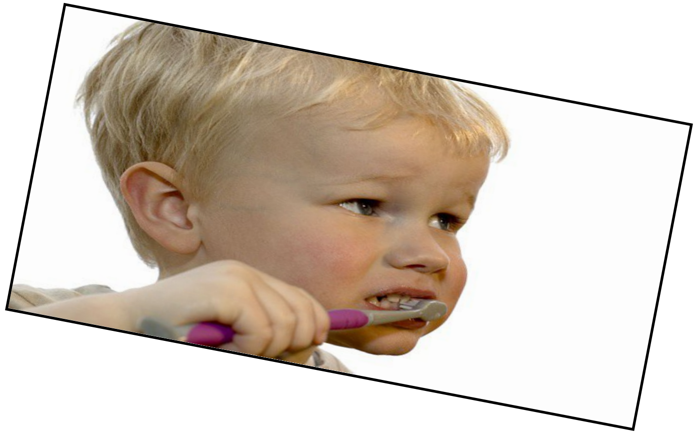
changes range of image function

$F$

Warping

$$G(\boldsymbol{x}) = F(h\{\boldsymbol{x}\})$$

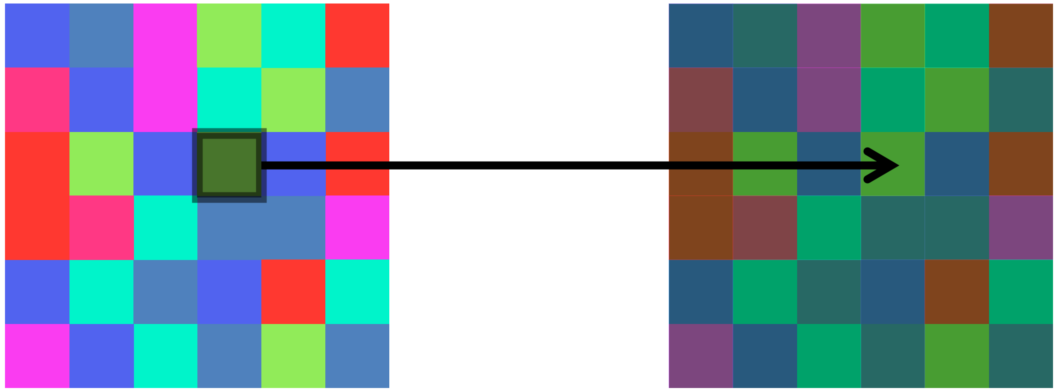
$G$



changes domain of image function

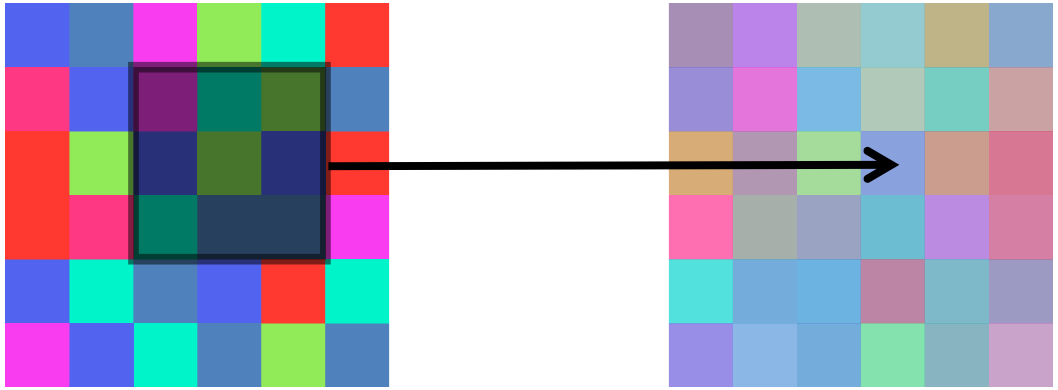
# What types of image filtering can we do?

Point Operation



point processing

Neighborhood Operation



“filtering”

# Point processing

# Pixel (Point) Processing

*Transformation* $T$ *of intensity* $f$ *at each pixel to intensity* $g$:

$$g(x,y) = T(f(x,y))$$

How would you implement these transformations?

# Examples of point processing

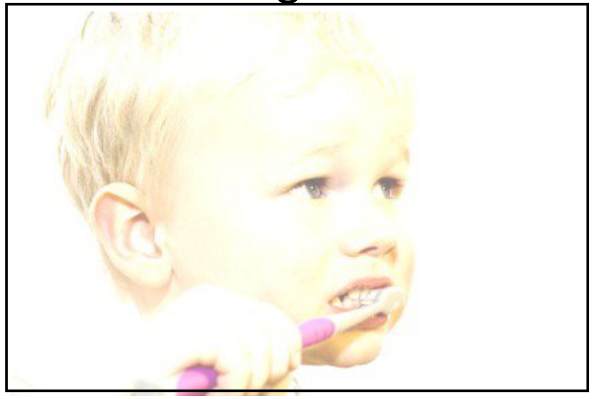
original

$f$

darken

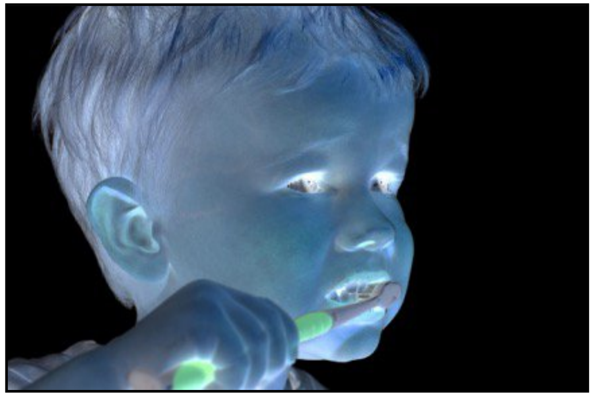
…

lighten



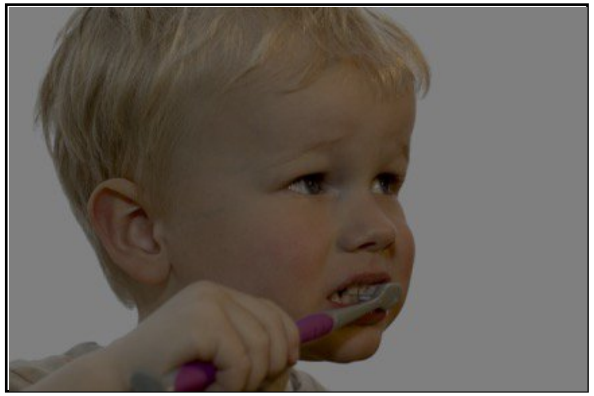
…

invert



…

lower contrast



…

raise contrast

…

# Examples of point processing

How would you implement these transformations?

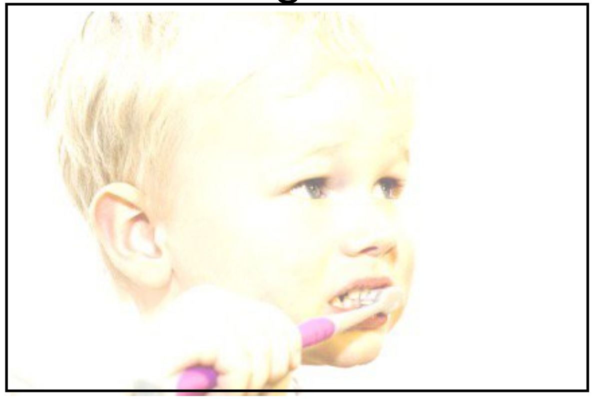
original

$f$

darken

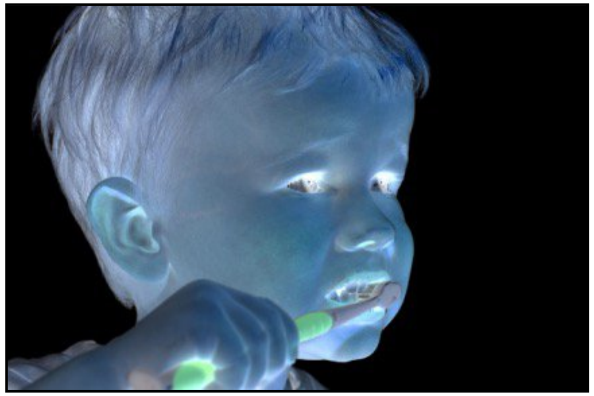
$f - 128$

lighten



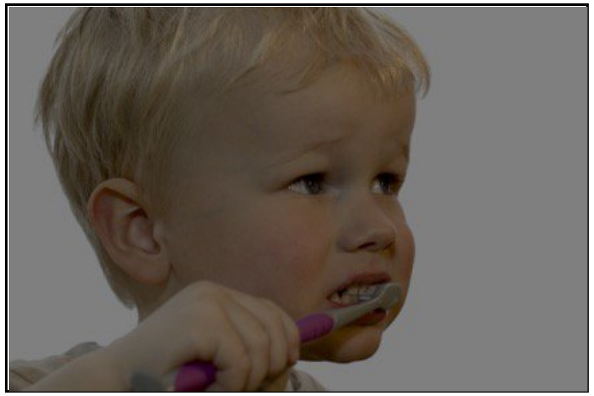
...

invert



...

lower contrast



...

raise contrast

...

How would you implement these transformations?

# Examples of point processing

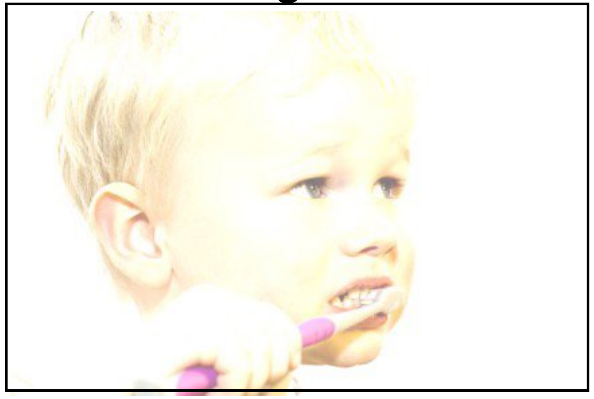
original

$f$

darken

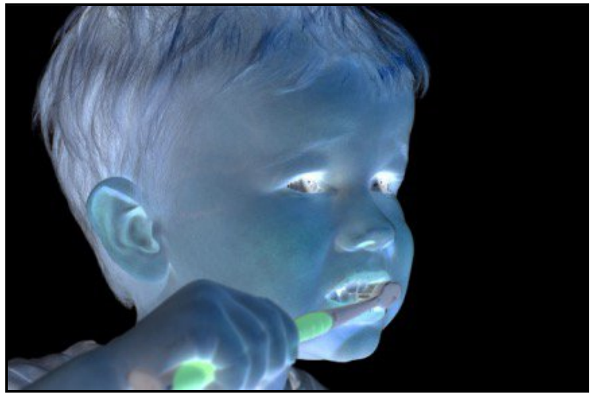
$f - 128$

lighten



$f + 128$

invert



…

lower contrast

…

raise contrast

…

How would you implement these transformations?

# Examples of point processing

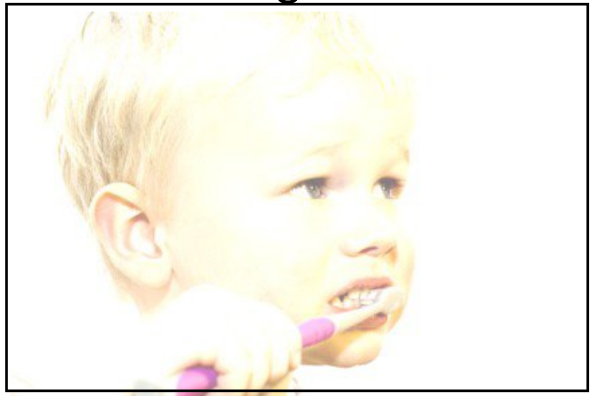
original

$f$

darken

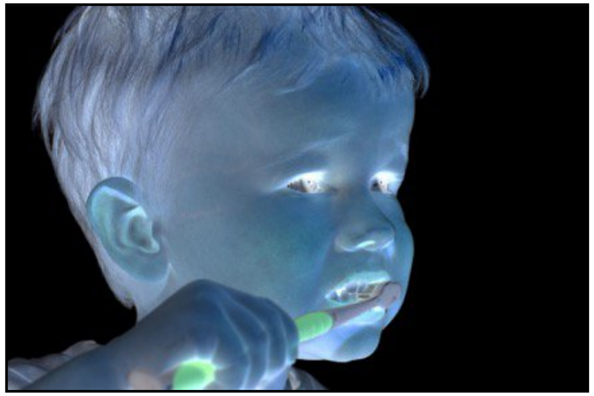
$f - 128$

lighten



$f + 128$

invert



$255 - f$

lower contrast

...

raise contrast

...

How would you implement these transformations?

# Examples of point processing

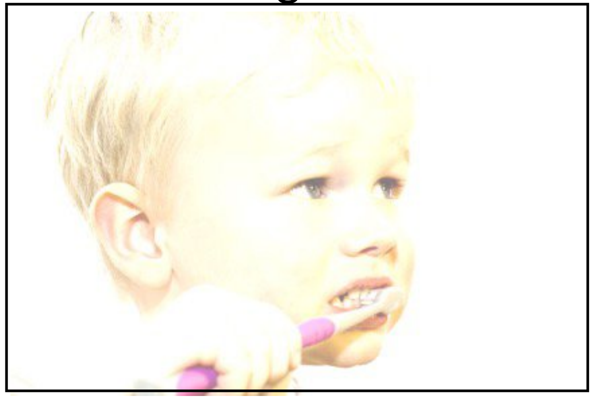
original

$f$

darken

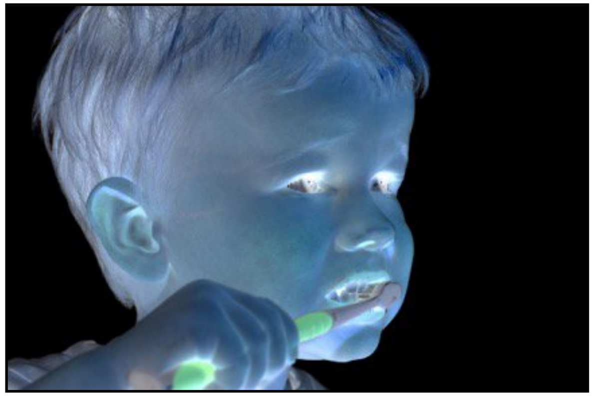
$f - 128$

lighten



$f + 128$

invert



$255 - f$

lower contrast

$f/2$

raise contrast

...

How would you implement these transformations?

# Examples of point processing

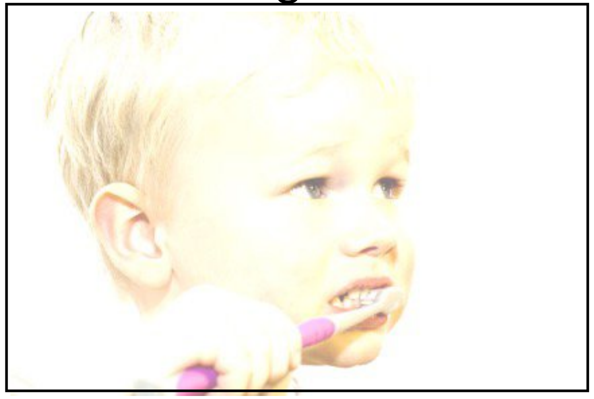
original

$f$

darken

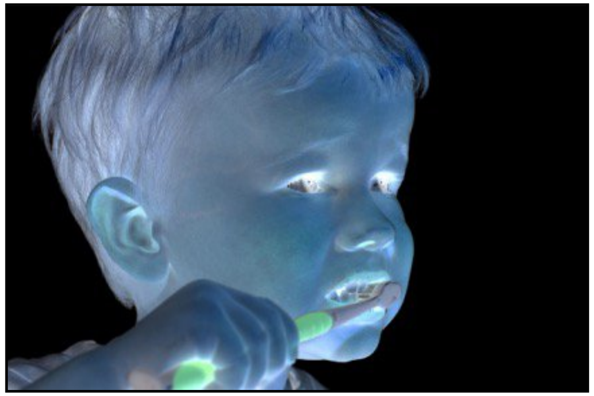
$f - 128$

lighten



$f + 128$

invert



$255 - f$

lower contrast

$f/2$

raise contrast

$f \times 2$

# Linear shift-invariant systems

# Linear shift-invariant system (LSIS)

$$f(x) \longrightarrow \boxed{\text{LSIS}} \longrightarrow g(x)$$

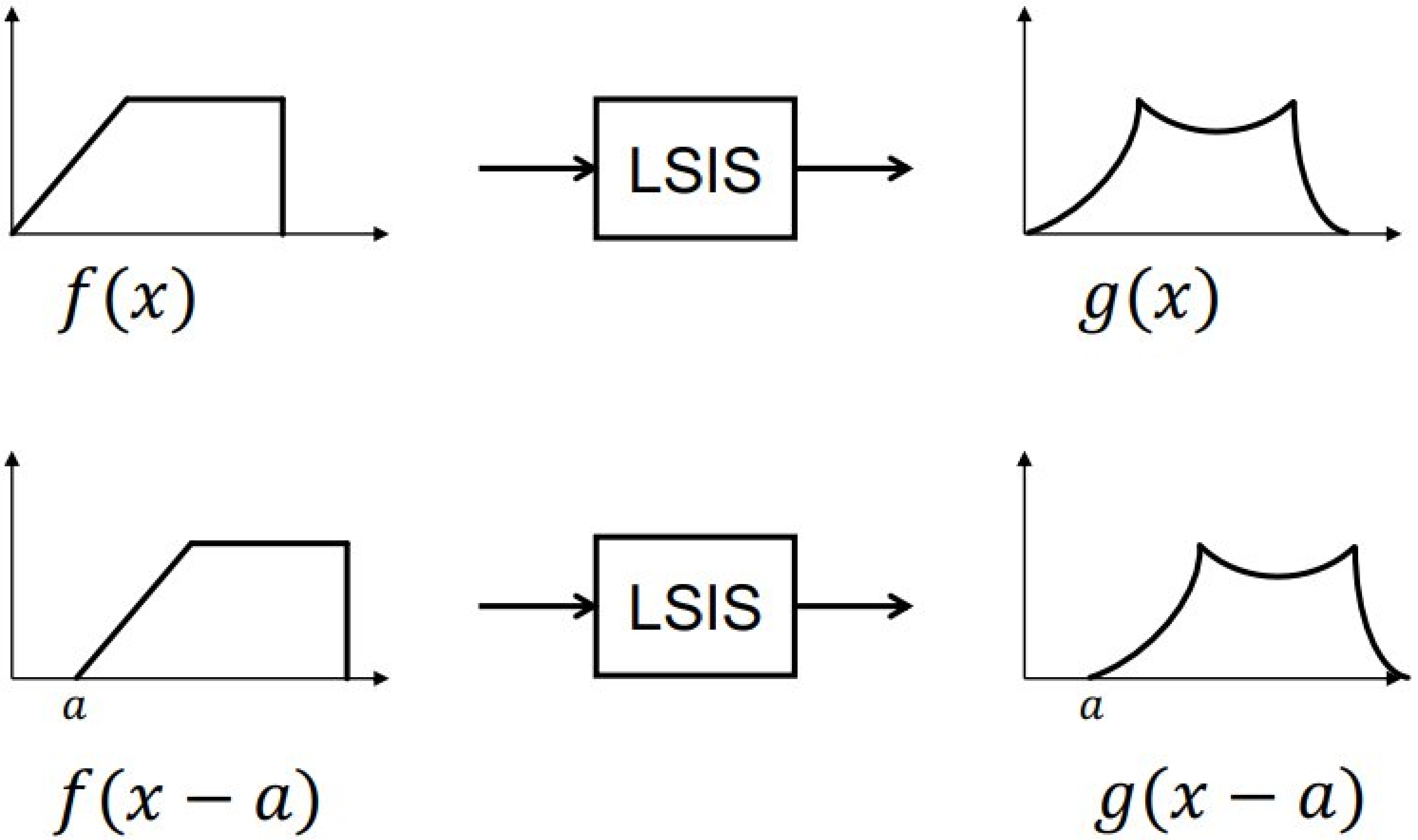
Study of Linear Shift Invariant Systems (LSIS) leads to useful image processing algorithms.

# LSIS : Linearity

$$f_1 \longrightarrow \boxed{\text{LSIS}} \longrightarrow g_1 \qquad\qquad f_2 \longrightarrow \boxed{\text{LSIS}} \longrightarrow g_2$$

$$\alpha f_1 + \beta f_2 \longrightarrow \boxed{\text{LSIS}} \longrightarrow \alpha g_1 + \beta g_2$$

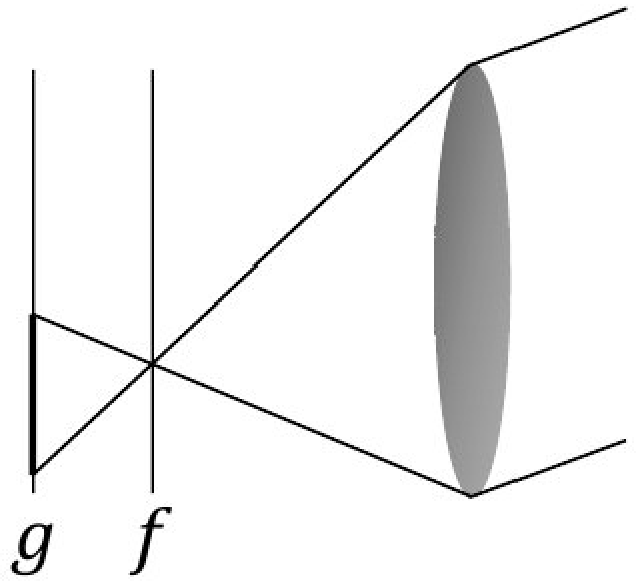
# LSIS: Shift Invariance



# Ideal Lens is an LSIS

Defocused Image (g) : Processed version of Focused Image (f)

- Linearity: Brightness variation
- Shift invariance: Scene movement




**Why This Matters**

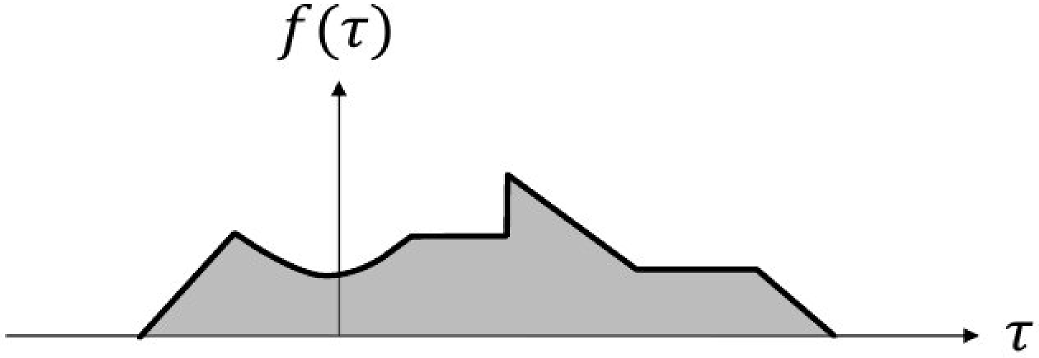
Any system that is both Linear and Shift Invariant can be represented as a **convolution** with some kernel!

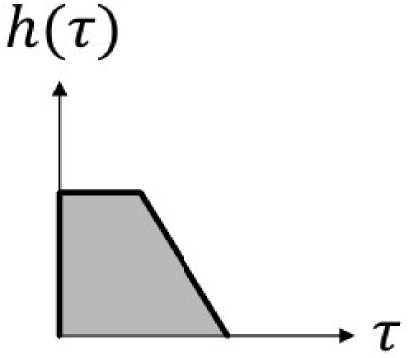
# Convolution

# Convolution

The convolution of two functions $\mathrm{f(x)}$ and $\mathrm{h(x)}$ is:

$$\boxed{g(x) = f(x) * h(x) = \int_{-\infty}^{\infty} f(\tau)h(x-\tau)\, d\tau}$$



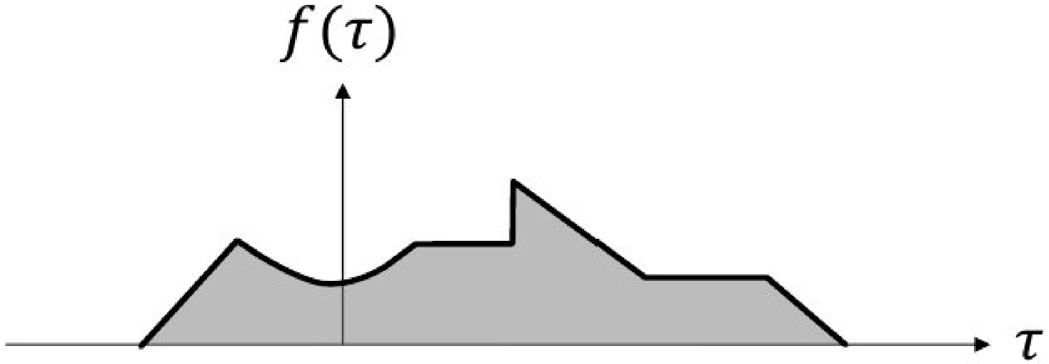



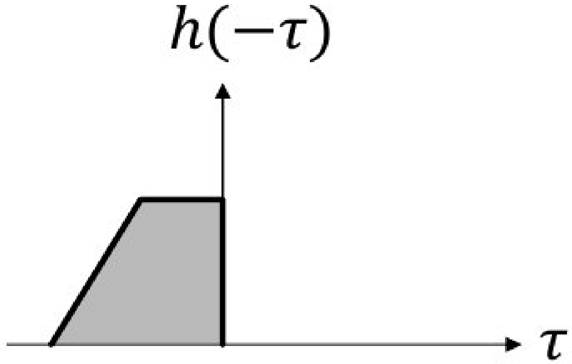
# Convolution

The convolution of two functions $f(x)$ and $h(x)$ is:

$$\boxed{g(x) = f(x) * h(x) = \int_{-\infty}^{\infty} f(\tau)h(x-\tau)\,d\tau}$$

**Step 1:** we take $h(\tau)$ and flip it about the vertical axis to get $h(-\tau)$



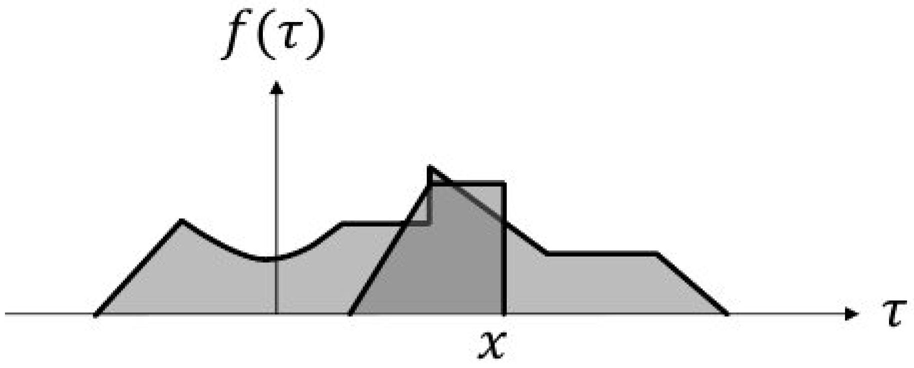



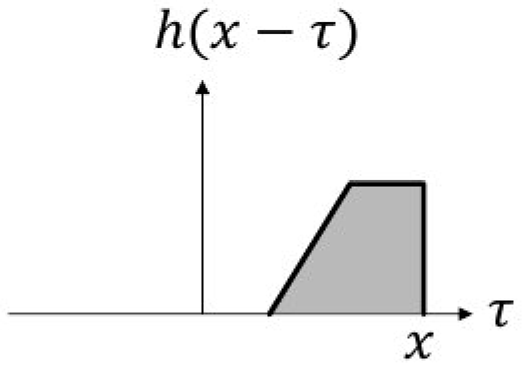
# Convolution

The convolution of two functions $f(x)$ and $h(x)$ is:

$$g(x) = f(x) * h(x) = \int_{-\infty}^{\infty} f(\tau)h(x-\tau)\, d\tau$$

**Step 2:** we shift $h(-\tau)$ by $x$ to get $h(x-\tau)$, which is then overlaid on $f(\tau)$



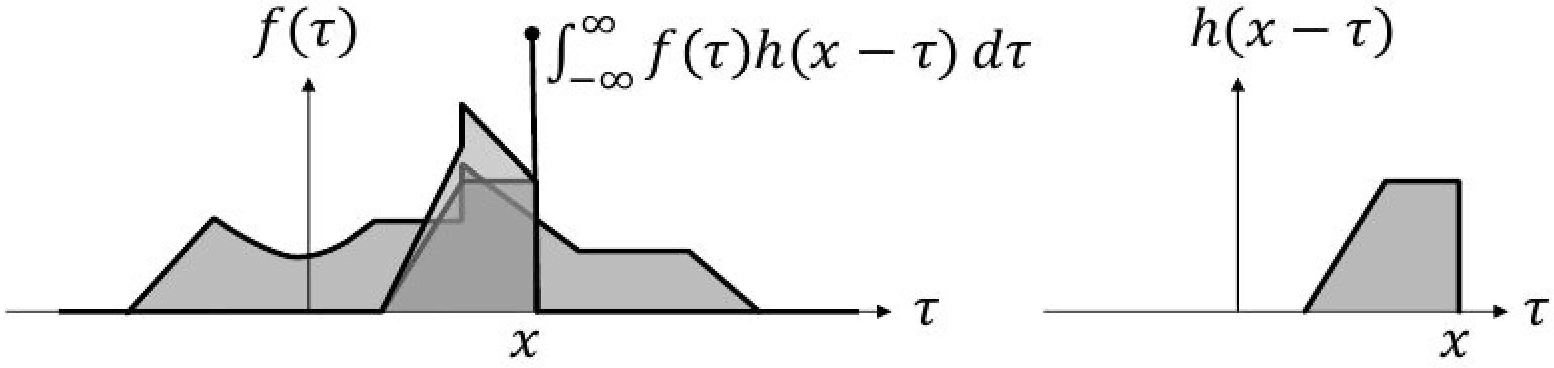



# Convolution

The convolution of two functions $f(x)$ and $h(x)$ is:

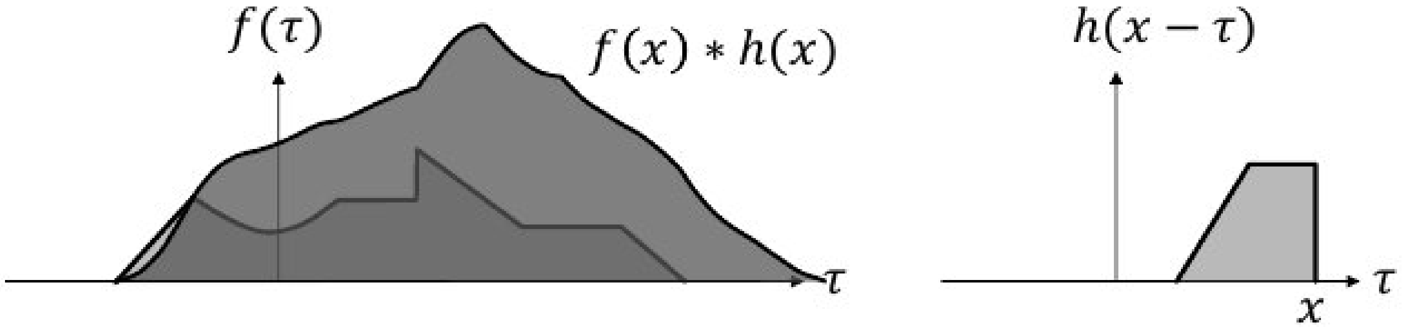
$$\boxed{g(x) = f(x) * h(x) = \int_{-\infty}^{\infty} f(\tau)h(x-\tau)\,d\tau}$$

**Step 3:** we take the product $f(\tau)h(x-\tau)$, and integrate it from -infinity to infinity



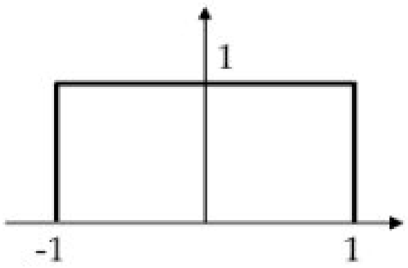
# Convolution

**Step 4:** We then vary the shift from minus infinity to plus infinity by sliding the function h(-$\tau$) over f($\tau$) from left to right.

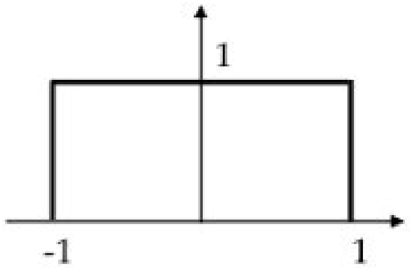



**LSIS implies Convolution and Convolution implies LSIS**

# Convolution: Example

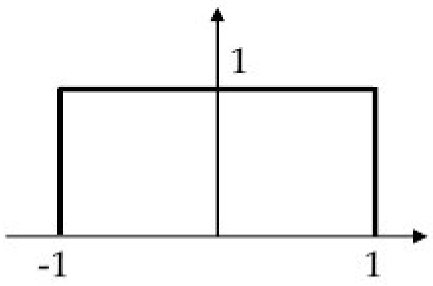



$f(x)$

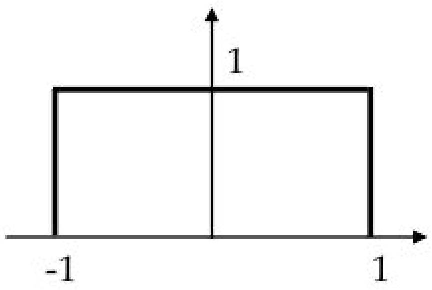



$h(x)$

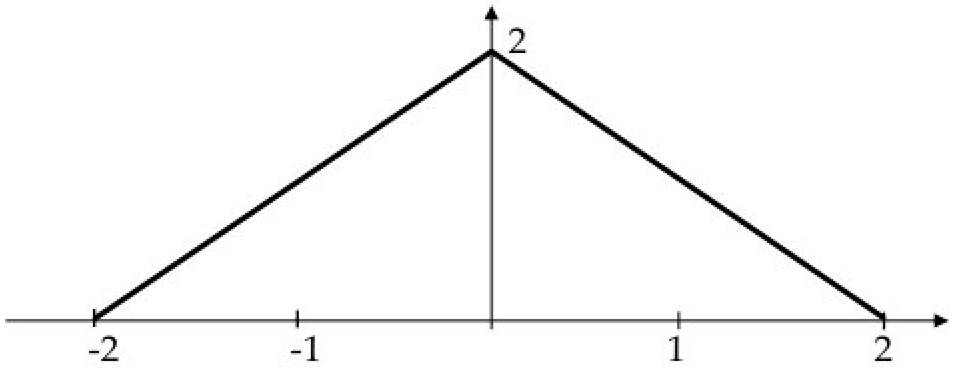
# Convolution: Example




$f(x)$




$h(x)$




$f(x) * h(x)$

# Convolution: Online Demo


https://phiresky.github.io/convolution-demo/

# Convolution is LSIS

Linearity:

Let: $g_1(x) = \int_{-\infty}^{\infty} f_1(\tau)h(x-\tau)\,d\tau$ and $g_2(x) = \int_{-\infty}^{\infty} f_2(\tau)h(x-\tau)\,d\tau$

Then:

$$\int_{-\infty}^{\infty} (\alpha f_1(\tau) + \beta f_2(\tau))h(x-\tau)\,d\tau$$

$$= \alpha \int_{-\infty}^{\infty} f_1(\tau)h(x-\tau)\,d\tau + \beta \int_{-\infty}^{\infty} f_2(\tau)h(x-\tau)\,d\tau$$

$$= \alpha g_1(x) + \beta g_2(x)$$

# Convolution is LSIS

Shift Invariance:

Let: $g(x) = \int_{-\infty}^{\infty} f(\tau)h(x-\tau)\,d\tau$

Then:

$$\int_{-\infty}^{\infty} f(\tau - a)h(x-\tau)\,d\tau$$

$$= \int_{-\infty}^{\infty} f(\mu)h(x-a-\mu)\,d\mu \quad \boxed{1} \quad \text{(Substituting } \mu = \tau - a\text{)}$$

$$= g(x-a)$$

# Can we find h?

$$f \longrightarrow \boxed{h} \longrightarrow g \qquad g(x) = \int_{-\infty}^{\infty} f(\tau)h(x-\tau)\,d\tau$$

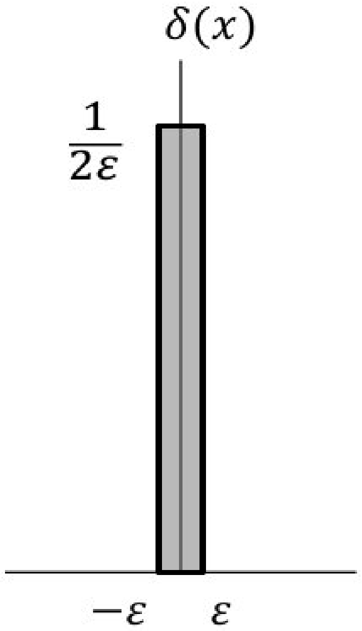
What input $f$ will produce output $g = h$ ?

$$h(x) = \int_{-\infty}^{\infty} ?\,(\tau)h(x-\tau)\,d\tau$$

# Unit Impulse Function

$$\delta(x) = \begin{cases} 1/2\varepsilon\,, & |x| \le \varepsilon \\ 0, & |x| > \varepsilon \end{cases}$$

$$\varepsilon \to 0$$

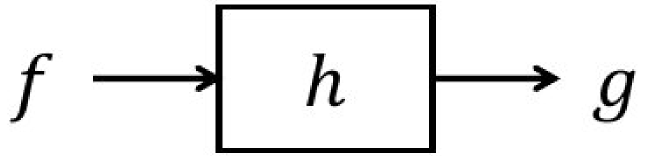


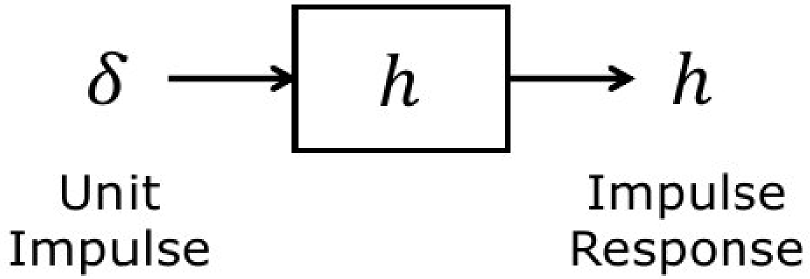

$$\int_{-\infty}^{\infty} \delta(\tau)\, d\tau = \frac{1}{2\varepsilon}.\,2\varepsilon = 1$$

$$\boxed{\int_{-\infty}^{\infty} \delta(\tau) b(x-\tau)\, d\tau = b(x)}$$

Sifting Property

# Impulse Response




$$g(x) = f(x) * h(x)$$

$$h(x) = \delta(x) * h(x)$$

$$g(x) = \int_{-\infty}^{\infty} f(\tau)h(x-\tau)\,d\tau$$

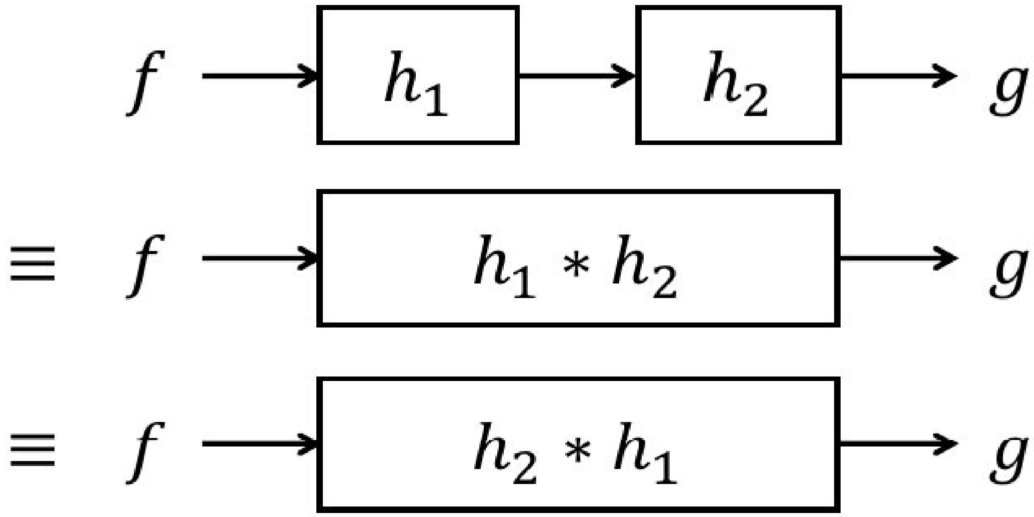
$$h(x) = \int_{-\infty}^{\infty} \delta(\tau)h(x-\tau)\,d\tau$$

# Properties of Convolution

Commutative $a * b = b * a$

Associative $(a * b) * c = a * (b * c)$

Cascaded System



# 2D Convolution

LSIS:

$$f(x,y) \longrightarrow \boxed{h(x,y)} \longrightarrow g(x,y)$$

Convolution:

$$g(x,y) = \iint_{-\infty}^{\infty} f(\tau,\mu)h(x-\tau,y-\mu)\,d\tau d\mu$$

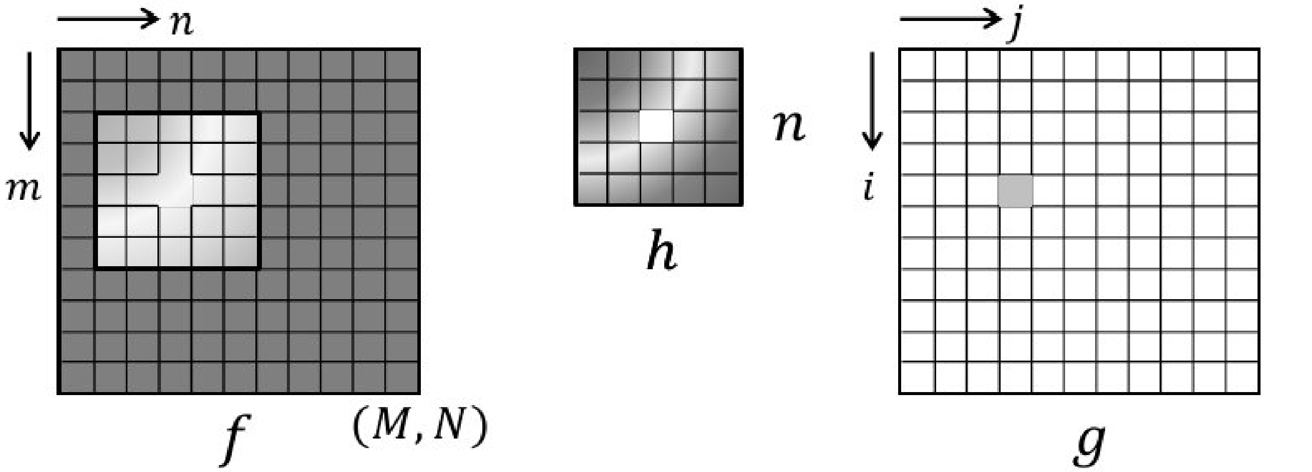
$$\boxed{1}$$

# Linear shift-invariant image filtering

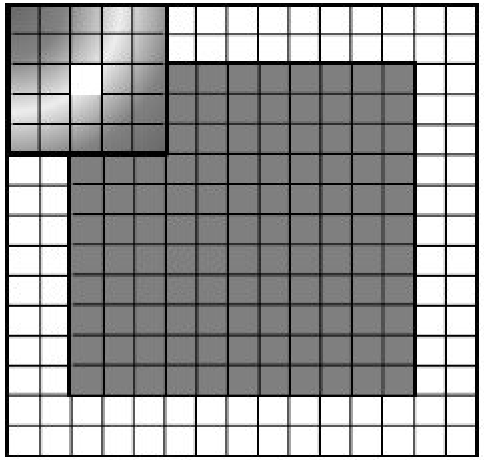
# Convolution with Discrete Images

$$f[i,j] \longrightarrow \boxed{h[i,j]} \longrightarrow g[i,j]$$

$$g[i,j] = \sum_{m=1}^{M} \sum_{n=1}^{N} f[m,n] \underbrace{h[i-m, j-n]}_{\text{"Mask," "Kernel," "Filter"}} \qquad \boxed{1}$$



# Border Problem



Solution:

- Ignore border
- Pad with constant value
- Pad with reflection

# Linear shift-invariant image filtering

- Replace each pixel by a linear combination of its neighbors (and possibly itself).
- The combination is determined by the filter’s kernel.
- The same kernel is shifted to all pixel locations so that all pixels use the same linear combination of their neighbors.

# Example: the box filter



- also known as the 2D rect (not rekt) filter
- also known as the square mean filter

kernel $h(i,j) = \frac{1}{9} \begin{array}{|c|c|c|} \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$

- replaces pixel with local average
- has smoothing (blurring) effect

# Let’s run the box filter

kernel

$$h(i,j) = \frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

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note that we assume that the kernel coordinates are centered

$$g[i,j] = \sum_{m=1}^{M} \sum_{n=1}^{N} f[m,n] \underbrace{h[i-m, j-n]}_{\text{"Mask," "Kernel," "Filter"}}$$

# Let’s run the box filter

kernel

$$h(i,j) = \frac{1}{9}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

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| | 0 | | | | | | | | |
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| | | | | | | | | | |

$$g[i,j] = \sum_{m=1}^{M} \sum_{n=1}^{N} f[m,n] \underbrace{h[i-m, j-n]}_{\text{“Mask,” “Kernel,” “Filter”}}$$

# Let’s run the box filter

kernel

$$h(i,j) = \frac{1}{9}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

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| | | | | | | | | | |
| | 0 | | | | | | | | |
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shift-invariant: as the pixel shifts, so does the kernel

$$g[i,j] = \sum_{m=1}^{M} \sum_{n=1}^{N} f[m,n] \underbrace{h[i-m, j-n]}_{\text{“Mask,” “Kernel,” “Filter”}}$$

# Let’s run the box filter

kernel

$$h(i,j) = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

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| | 0 | 10 | | | | | | | |
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$$g[i,j] = \sum_{m=1}^{M}\sum_{n=1}^{N} f[m,n]\underbrace{h[i-m,j-n]}_{\text{“Mask,” “Kernel,” “Filter”}}$$

# Let’s run the box filter

kernel

$$h(i,j) = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

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| | 0 | 10 | | | | | | | |
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$$g[i,j] = \sum_{m=1}^{M}\sum_{n=1}^{N} f[m,n]\underbrace{h[i-m,j-n]}_{\text{“Mask,” “Kernel,” “Filter”}}$$

# Let’s run the box filter

kernel

$$h(i,j) = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

| | | | | | | | | | |
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| | 0 | 10 | 20 | | | | | | |
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$$g[i,j] = \sum_{m=1}^{M}\sum_{n=1}^{N} f[m,n]\underbrace{h[i-m,j-n]}_{\text{“Mask,” “Kernel,” “Filter”}}$$

# Let's run the box filter

$$h(i,j) = \frac{1}{9}$$

kernel

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

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| | 0 | 10 | 20 | | | | | | |
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$$g[i,j] = \sum_{m=1}^{M}\sum_{n=1}^{N} f[m,n]\underbrace{h[i-m,j-n]}_{\text{“Mask,” “Kernel,” “Filter”}}$$

# Let’s run the box filter

kernel

$$h(i,j) = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

| | | | | | | | | | |
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| | 0 | 10 | 20 | 30 | | | | | |
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$$g[i,j] = \sum_{m=1}^{M}\sum_{n=1}^{N} f[m,n]\underbrace{h[i-m,j-n]}_{\text{“Mask,” “Kernel,” “Filter”}}$$

# Let’s run the box filter

kernel

$$h(i,j) = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

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| | 0 | 10 | 20 | 30 | 30 | | | | |
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$$g[i,j] = \sum_{m=1}^{M}\sum_{n=1}^{N} f[m,n]\underbrace{h[i-m,j-n]}_{\text{“Mask,” “Kernel,” “Filter”}}$$

# Let’s run the box filter

kernel

$$h(i,j) = \frac{1}{9}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

| | | | | | | | | | |
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| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | | | |
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$$g[i,j] = \sum_{m=1}^{M}\sum_{n=1}^{N} f[m,n]\underbrace{h[i-m,j-n]}_{\text{“Mask,” “Kernel,” “Filter”}}$$

# Let's run the box filter

kernel

$$h(i,j) = \frac{1}{9}\begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline\end{array}$$

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

| | | | | | | | | | |
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| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | | |
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$$g[i,j] = \sum_{m=1}^{M}\sum_{n=1}^{N} f[m,n]\underbrace{h[i-m,j-n]}_{\text{“Mask,” “Kernel,” “Filter”}}$$

# Let’s run the box filter

kernel

$$h(i,j) = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

| | | | | | | | | | |
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| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
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$$g[i,j] = \sum_{m=1}^{M} \sum_{n=1}^{N} f[m,n] \underbrace{h[i-m, j-n]}_{\text{“Mask,” “Kernel,” “Filter”}}$$

# Let’s run the box filter

kernel

$$h(i,j) = \frac{1}{9}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

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| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 0 | | | | | | | | |
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$$g[i,j] = \sum_{m=1}^{M} \sum_{n=1}^{N} f[m,n] \underbrace{h[i-m, j-n]}_{\text{“Mask,” “Kernel,” “Filter”}}$$

# Let's run the box filter

kernel

$$h(i,j) = \frac{1}{9}\begin{bmatrix}1 & 1 & 1\\ 1 & 1 & 1\\ 1 & 1 & 1\end{bmatrix}$$

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 0 | 20 | | | | | | | |
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$$g[i,j] = \sum_{m=1}^{M}\sum_{n=1}^{N} f[m,n]\underbrace{h[i-m,j-n]}_{\text{"Mask," "Kernel," "Filter"}}$$

# Let's run the box filter

kernel

$$h(i,j) = \frac{1}{9}\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 0 | 20 | 40 | | | | | | |
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$$g[i,j] = \sum_{m=1}^{M}\sum_{n=1}^{N} f[m,n]\underbrace{h[i-m,j-n]}_{\text{"Mask," "Kernel," "Filter"}}$$

# Let’s run the box filter

kernel

$$h(i,j) = \frac{1}{9}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 0 | 20 | 40 | 60 | 60 | 60 | 40 | 20 | |
| | 0 | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

$$g[i,j] = \sum_{m=1}^{M}\sum_{n=1}^{N} f[m,n]\underbrace{h[i-m,j-n]}_{\text{“Mask,” “Kernel,” “Filter”}}$$

# Let’s run the box filter

kernel

$$h(i,j) = \frac{1}{9}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 0 | 20 | 40 | 60 | 60 | 60 | 40 | 20 | |
| | 0 | 30 | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |
| | | | | | | | | | |

$$g[i,j] = \sum_{m=1}^{M} \sum_{n=1}^{N} f[m,n] \underbrace{h[i-m, j-n]}_{\text{“Mask,” “Kernel,” “Filter”}}$$

# Let’s run the box filter

kernel

$$h(i,j) = \frac{1}{9}$$

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 0 | 20 | 40 | 60 | 60 | 60 | 40 | 20 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 20 | 30 | 50 | 50 | 60 | 40 | 20 | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 10 | 10 | 10 | 10 | 0 | 0 | 0 | 0 | |
| | 10 | | | | | | | | |
| | | | | | | | | | |

$$g[i,j] = \sum_{m=1}^{M}\sum_{n=1}^{N} f[m,n]\underbrace{h[i-m,j-n]}_{\text{“Mask,” “Kernel,” “Filter”}}$$

# Let’s run the box filter

$$h(i,j) = \frac{1}{9}$$

kernel

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 0 | 20 | 40 | 60 | 60 | 60 | 40 | 20 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 20 | 30 | 50 | 50 | 60 | 40 | 20 | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 10 | 10 | 10 | 10 | 0 | 0 | 0 | 0 | |
| | 10 | 10 | 10 | 10 | 0 | 0 | 0 | 0 | |
| | | | | | | | | | |

$$g[i,j] = \sum_{m=1}^{M} \sum_{n=1}^{N} f[m,n] \underbrace{h[i-m, j-n]}_{\text{“Mask,” “Kernel,” “Filter”}}$$

# … and the result is

$$h(i,j) = \frac{1}{9}$$

kernel

| | | |
|---|---|---|
| 1 | 1 | 1 |
| 1 | 1 | 1 |
| 1 | 1 | 1 |

image $f(i,j)$

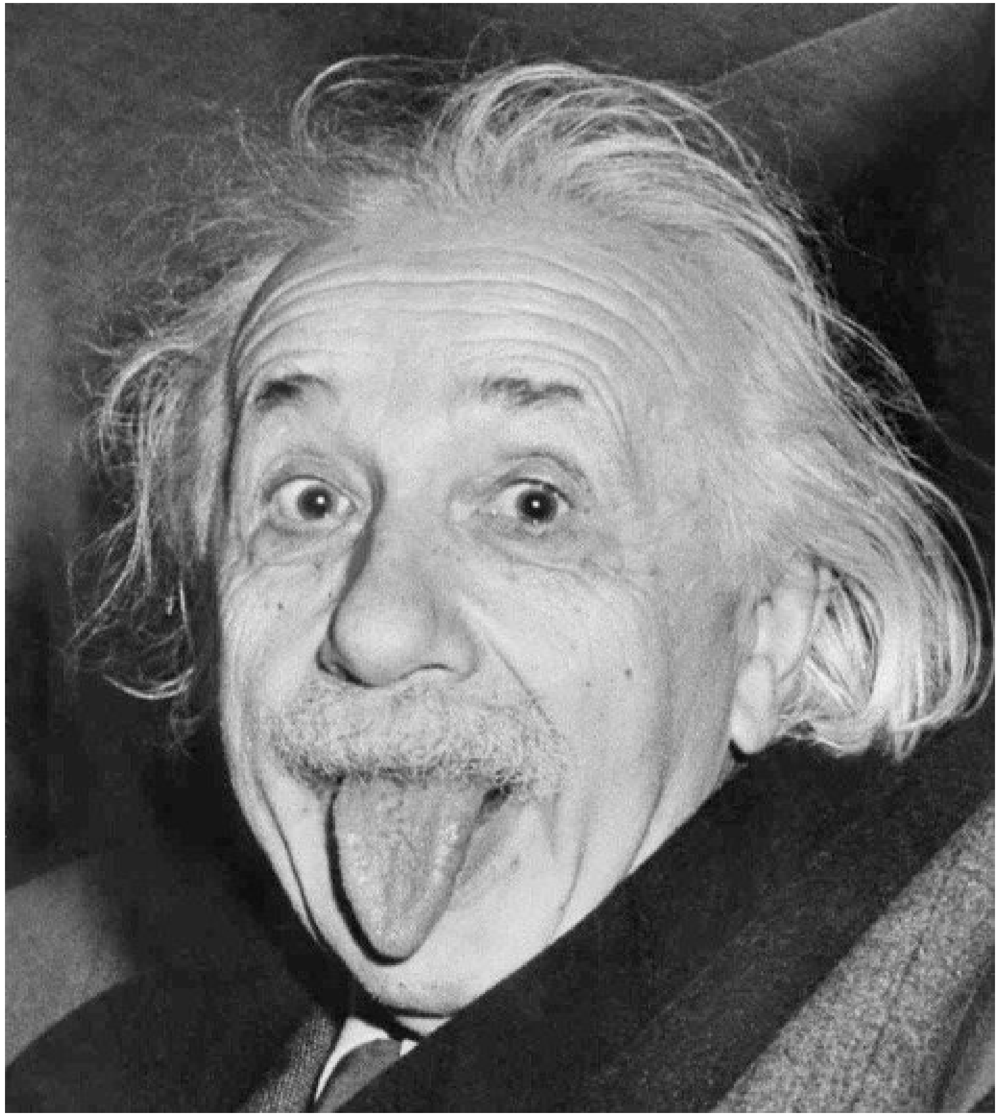
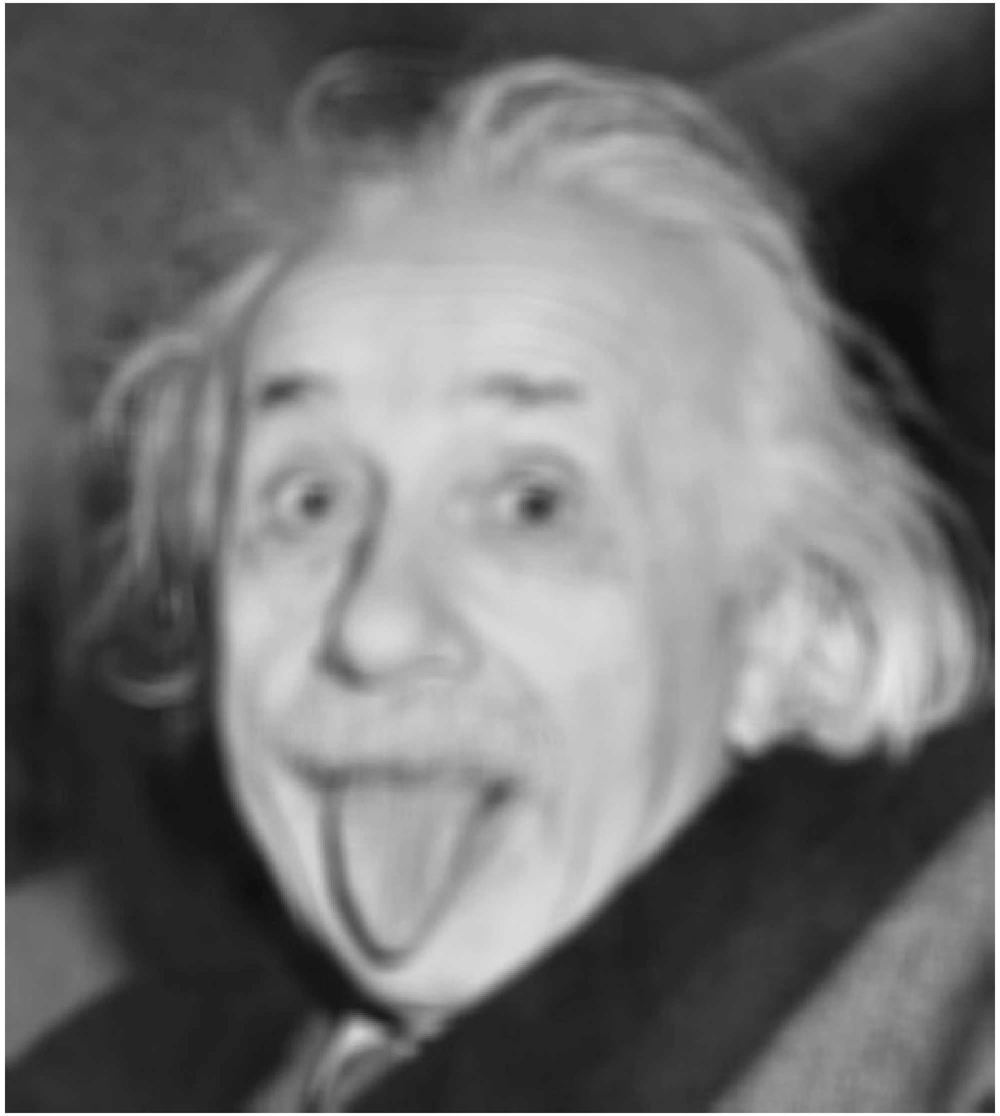
| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 0 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 90 | 90 | 90 | 90 | 90 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 90 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |

output $g(i,j)$

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | | | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 0 | 20 | 40 | 60 | 60 | 60 | 40 | 20 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 30 | 50 | 80 | 80 | 90 | 60 | 30 | |
| | 0 | 20 | 30 | 50 | 50 | 60 | 40 | 20 | |
| | 0 | 10 | 20 | 30 | 30 | 30 | 20 | 10 | |
| | 10 | 10 | 10 | 10 | 0 | 0 | 0 | 0 | |
| | 10 | 10 | 10 | 10 | 0 | 0 | 0 | 0 | |
| | | | | | | | | | |

$$g[i,j] = \sum_{m=1}^{M}\sum_{n=1}^{N} f[m,n]\underbrace{h[i-m,j-n]}_{\text{"Mask," "Kernel," "Filter"}}$$

# Example: the box filter

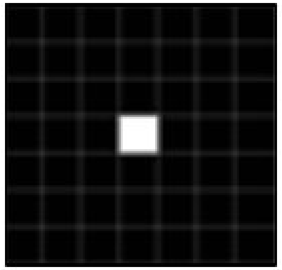
 

# Example: the impulse filter

Input

Output


$*$



$=$

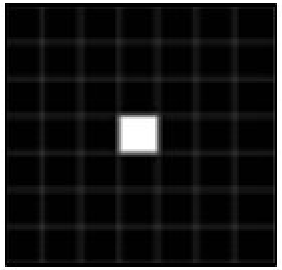
$f(x, y)$

$\delta(x, y)$

# Example: the impulse filter

Input


$$*$$



$$=$$

Output

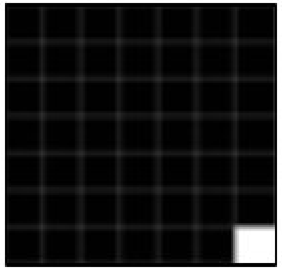

$f(x, y)$ $\quad$ $\delta(x, y)$ $\quad$ $f(x, y)$

# Example: the impulse filter

Input

Output


$*$



$=$

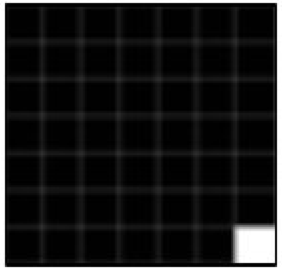
$f(x, y)$

$\delta(x - u, y - v)$

# Example: Image shift

Input


$*$



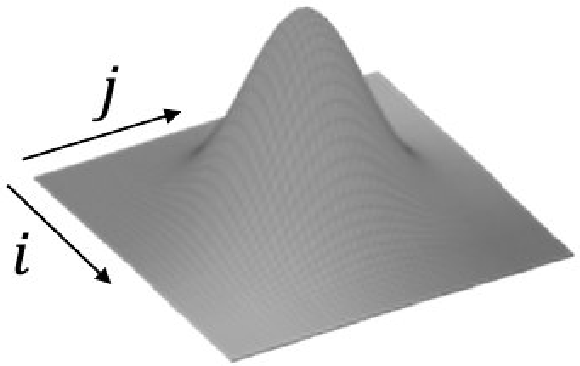
$=$

Output

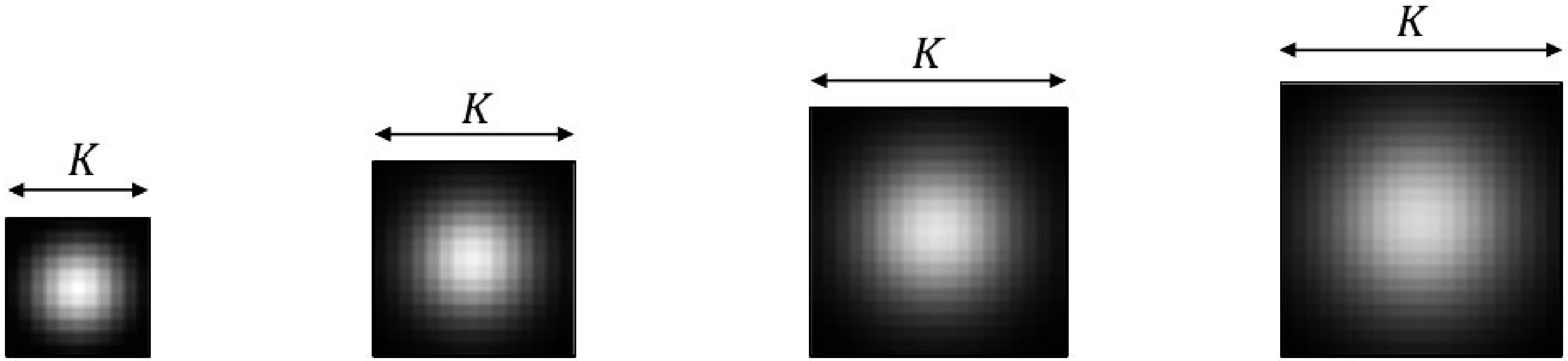
$f(x, y)$ $\delta(x - u, y - v)$ $f(x - u, y - v)$

# Smoothing With the Gaussian Filter

$$n_\sigma[i,j] = \frac{1}{2\pi\sigma^2} e^{-\frac{1}{2}\left(\frac{i^2+j^2}{\sigma^2}\right)} \quad \boxed{1}$$




$\sigma^2$: Variance



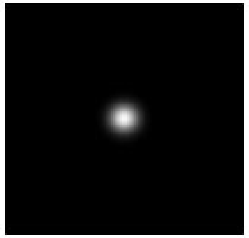

$\sigma = 2$ $\quad$ $\sigma = 3$ $\quad$ $\sigma = 4$ $\quad$ $\sigma = 5$

Rule of thumb: Set kernel size $K \approx 2\pi\sigma$

# Smoothing With the Gaussian Filter

Input

$*$



$=$

Output

$\sigma = 4$

$f(x, y)$ $\quad$ $n_4(x, y)$ $\quad$ $g(x, y)$

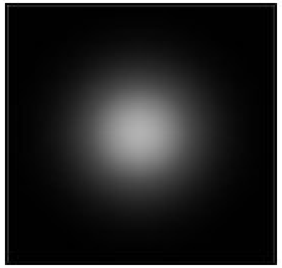
Larger the kernel (or σ), more the blurring

# Smoothing With the Gaussian Filter

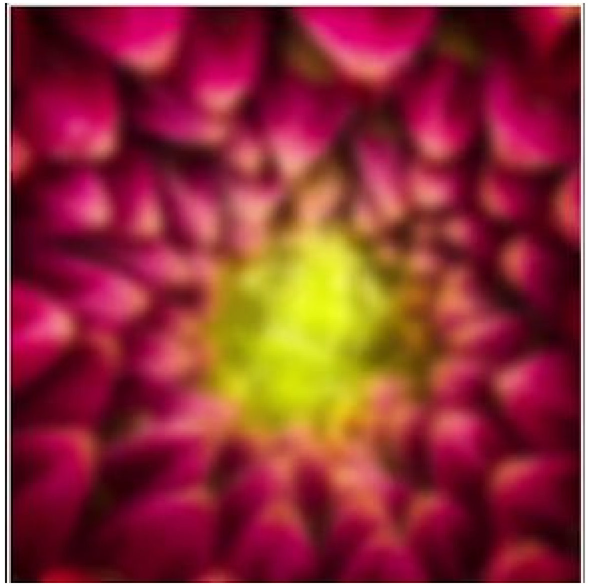
Input

Output

$*$



$=$



$\sigma = 16$

$f(x, y)$

$n_4(x, y)$

$g(x, y)$

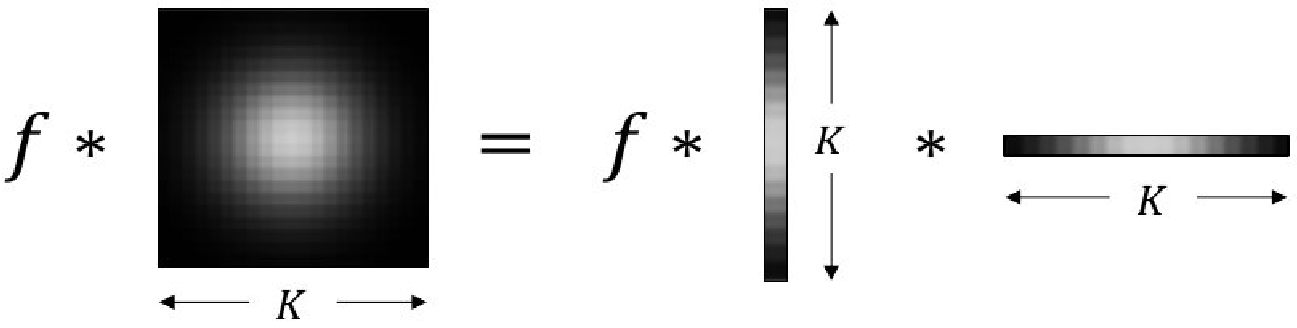
Larger the kernel (or σ), more the blurring

# Gaussian Smoothing is Separable

$$g[i,j] = \frac{1}{2\pi\sigma^2} \sum_{m=1}^{K} \sum_{n=1}^{K} e^{-\frac{1}{2}\left(\frac{m^2+n^2}{\sigma^2}\right)} f[i-m, j-n]$$

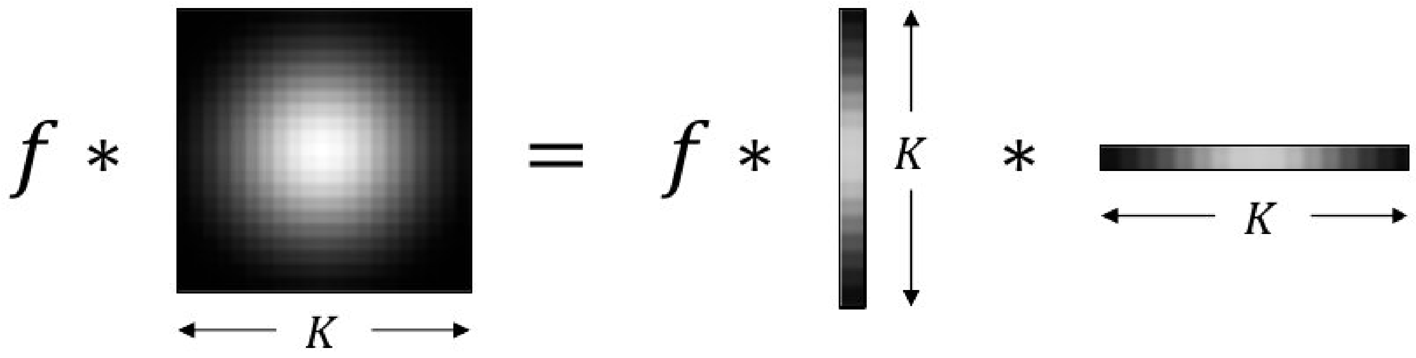
$$g[i,j] = \frac{1}{2\pi\sigma^2} \sum_{m=1}^{K} e^{-\frac{1}{2}\left(\frac{m^2}{\sigma^2}\right)} . \sum_{n=1}^{K} e^{-\frac{1}{2}\left(\frac{n^2}{\sigma^2}\right)} f[i-m, j-n]$$

Using One 2D Gaussian Filter ≡ Using Two 1D Gaussian Filters



# Gaussian Smoothing is Separable

Using One 2D Gaussian Filter ≡ Using Two 1D Gaussian Filters




Which one is faster? Why?

| | |
|---|---|
| $K^2$ Multiplications | $2K$ Multiplications |
| $K^2 - 1$ Additions | $2(K-1)$ Additions |

# Other filters

input

filter

$$\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 2 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} - \frac{1}{9} \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

output

**?**

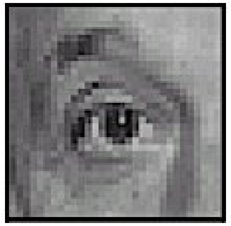
# Sharpening filter

input

filter

$$\begin{array}{|c|c|c|}\hline 0 & 0 & 0 \\ \hline 0 & 2 & 0 \\ \hline 0 & 0 & 0 \\ \hline \end{array} - \frac{\mathbf{1}}{\mathbf{9}} \begin{array}{|c|c|c|}\hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline 1 & 1 & 1 \\ \hline \end{array}$$

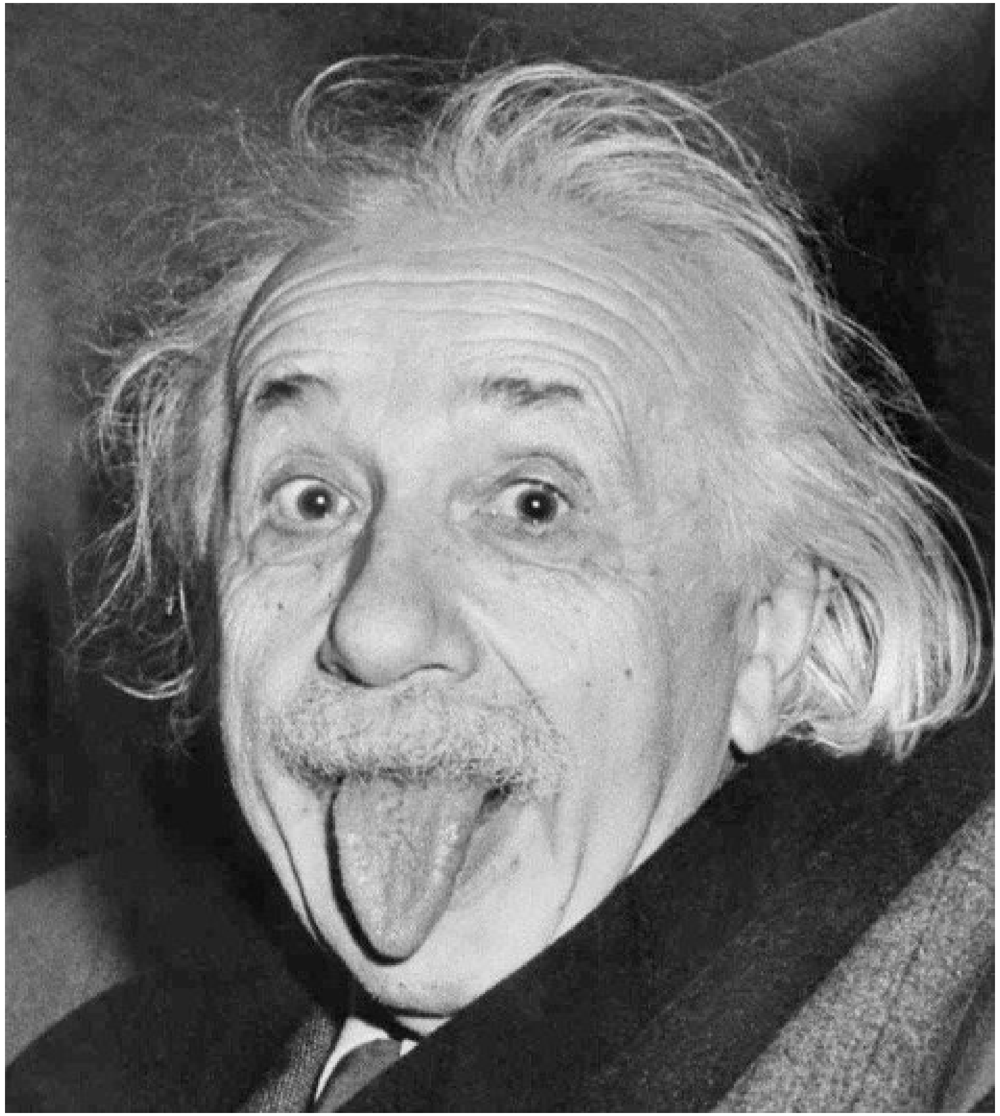
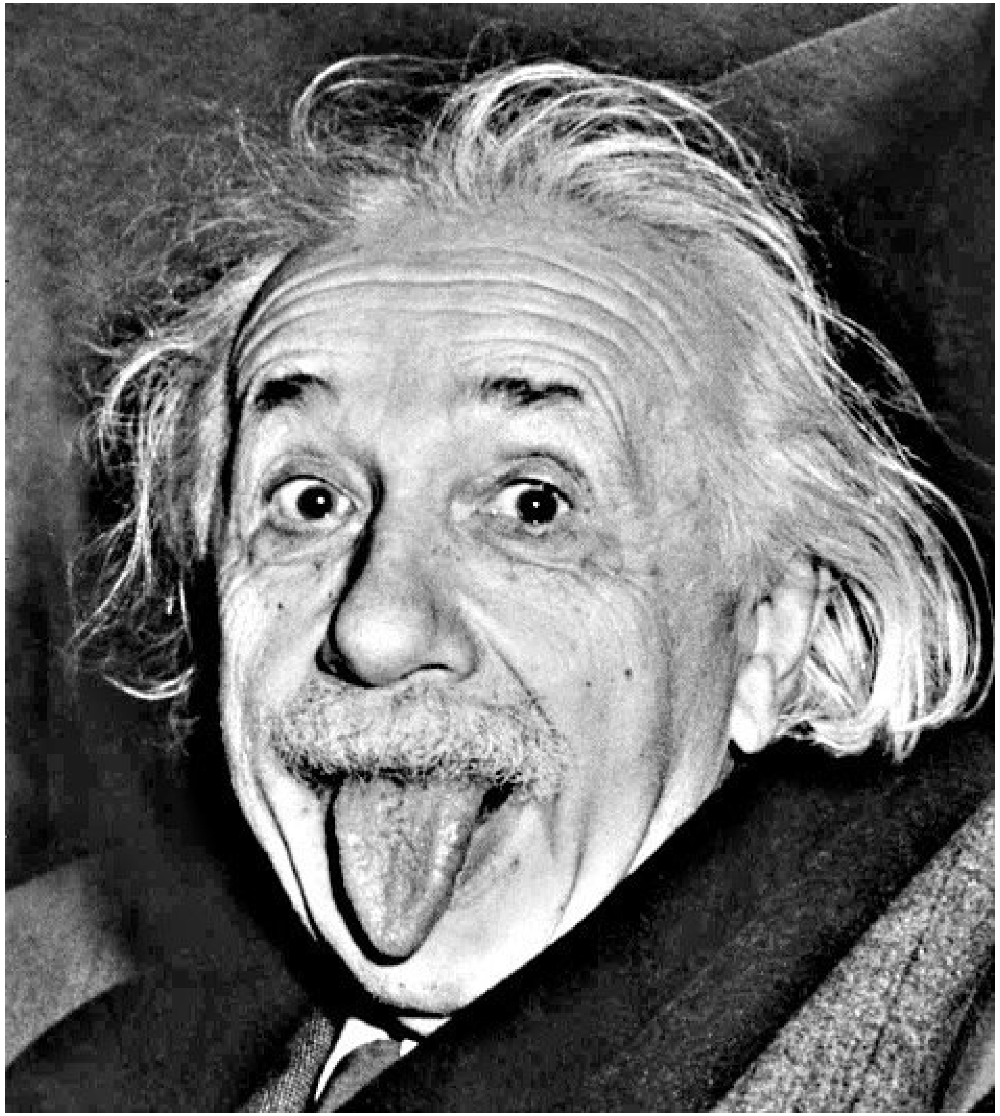
output



sharpening

- do nothing for flat areas (no change where intensity is constant)
- stress intensity peaks (enhance edges and sharp transitions)

# Sharpening examples

# Sharpening examples

# Sharpening examples